Variable is generally acknowledged as a difficult concept for students to grasp, but one that can be developed meaningfully over time (White & Mitchelmore, 1993). Definitions like these are usually found early in the sequence of textbook material intended to develop algebraic concepts. Without exploring the correctness or otherwise of the quoted definitions, they are not likely to contribute much to students' understanding of the concept. While language is crucial in the development of the concept of variable, the multi-faceted nature of the concept makes the use of a formal definition problematic.

Learning with definitions

The definition is an important language form in the register of mathematics. Students need to understand the structure of a definition so that they can make sense of the definitions they encounter and so that they can construct their own definitions as part of organising their thoughts about the concepts they have explored. However, I am suggesting in this article that there are many mathematical concepts for which the use of a single formal definition is not helpful in developing students' understanding of the concepts. Rather, more might be gained by having students use language, both mathematical and everyday, in some of the other language tasks that have been suggested (for example, Shield & Swinson, 1997).

Reference

Shield, M. J. & Swinson, K. V. (1997). Encouraging learning in mathematics through writing. *The Australian Mathematics Teacher, 53* (1), 4–9.

White, P. & Mitchelmore, M. (1993). Aiming for variable understanding. *The Australian Mathematics Teacher, 49* (4), 31–33.

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fraction

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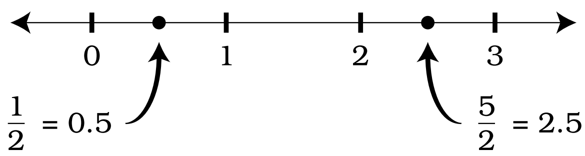
Fractions and decimals are often viewed by students as two separate sets of numbers rather than different representations of the same set of numbers. Often we reinforce this misconception with instruction that first focuses on computations with fractions only, or computations with decimals only. We can help students develop a clearer understanding of the concept of 'representation' of a number as a fraction or as a decimal by showing both names for a number on the number line;



To accomplish this, students need to be able to find the equivalent fraction for a decimal representation of a number and vice versa. Thus, we become familiar with the fractional representation of decimals, writing each decimal as a fraction whose denominator is a power of ten. Consider the following examples:

1. $0.1 = \dfrac{1}{10}$
2. $0.12 = \dfrac{12}{10^2} = \dfrac{12}{100}$
3. $0.123 = \dfrac{123}{10^3} = \dfrac{123}{1000}$
4. $0.1234 = \dfrac{1234}{10^4} = \dfrac{1234}{10000}$

When the decimals are non-terminating, but repeating, there is a procedure sometimes taught in junior high school to find its frac-

s and decimals

tional representation. (See also Beswick in this edition. Ed.) Consider the following examples:

1. 0.11111...

$$\begin{aligned}&\text{Let } x = 0.11111...\\&\text{then } 10x = 1.11111...\\&\text{so } 10x - x = 1\\&\qquad 9x = 1\\&\qquad x = \frac{1}{9}\end{aligned}$$

2. 0.12121212...

$$\begin{aligned}&\text{Let } x = 0.12121212...\\&\text{then } 100x = 12.121212...\\&\text{so } 100x - x = 12\\&\qquad 99x = 12\\&\qquad x = \frac{12}{99}\end{aligned}$$

3. 0.123123123...

$$\begin{aligned}&\text{Let } x = 0.123123123...\\&\text{then } 1000x = 123.123123123...\\&\text{so } 100x - x = 123\\&\qquad 999x = 123\\&\qquad x = \frac{123}{999}\end{aligned}$$

4. 0.49999...

$$\begin{aligned}&\text{Let } x = 0.49999...\\&\text{then } 10x = 4.9999...\\&\text{so } 10x - x = 4.5\\&\qquad 9x = 4.5\\&\qquad 90x = 45\\&\qquad x = \frac{45}{90}\\&\qquad x = \frac{1}{2}\end{aligned}$$

Working from fractions to the decimal representation, we simply divide the numerator by the denominator. Consider the following examples:

1. $\frac{1}{4} = 0.25$
2. $\frac{1}{8} = 0.125$
3. $\frac{1}{11} = 0.090909\ldots = 0.\overline{09}$
4. $\frac{1}{7} = 0.\overline{142857}$

When a sequence of digits repeats, this sequence is called the repetend and we indicate this by placing a bar over those digits. So, a fraction is a number of the form $\frac{a}{b}$ where a and b are whole numbers, and b ≠ 0. A fraction has a decimal representation that can be found by dividing a (the numerator) by b (the denominator).

The final step to showing students that fractions and decimals are just different ways to represent the same point on the number line is an investigation whereby students perform side-by-side computations that also provide equivalent results. Thus, if we add two fractions and their respective decimal representations, the sums should agree. Consider the next examples:

1.

$$\begin{array}{rcl} & \dfrac{12}{99} & = \quad 0.121212...\\ + & \dfrac{34}{99} & = \quad 0.343434...\\ \hline & \dfrac{46}{99} & = \quad 0.464646...\end{array}$$

2.

$$\begin{array}{rcl} \frac{61}{99} & = & 0.616161\ldots \\ + \quad \frac{52}{99} & = & 0.525252\ldots \\ \hline 1\frac{14}{99} & = & 1.141414\ldots \end{array}$$

Note that we are able to do this because of the infinite repetition of the repetend. This makes a nice connection between the two representations that students can explore with more commonly used fractions like

$$\frac{1}{3}+\frac{1}{6}=0.\overline{3}+0.1\overline{6}$$

recalling that

$$0.499\ldots=\frac{1}{2}$$

The results would hold true for subtraction of two fractions and their respective decimal representations. Consider the next examples:

1.

$$\begin{array}{rcl} \frac{54}{99} & = & 0.545454\ldots \\ - \quad \frac{31}{99} & = & 0.313131\ldots \\ \hline \frac{23}{99} & = & 0.232323\ldots \end{array}$$

2.

$$\begin{array}{rcl} \frac{41}{99} & = & 0.414141\ldots \\ - \quad \frac{27}{99} & = & 0.272727\ldots \\ \hline \frac{14}{99} & = & 0.141414\ldots \end{array}$$

However, we get into trouble trying to multiply (or divide) two decimal representations with infinitely recurring repetends. Students can, however, multiply and divide terminating decimals and their equivalent fractions. Consider the next two examples:

1.

$$\begin{array}{rcl} \frac{1}{4} & = & 0.25 \\ \times \quad \frac{1}{5} & = & 0.2 \\ \hline \frac{1}{20} & = & 0.05 \end{array}$$

2.

$$\frac{1}{8}=0.125$$

$$\frac{1}{4}=0.25$$

$$\frac{1}{8}\div\frac{1}{4}=\frac{1}{2}$$

$$0.125\div 0.25=0.5$$

Lastly, students can also multiply (or divide) both representations by whole numbers and notice that the results agree. Consider the following two examples:

1.

$$\frac{1}{6}=0.16666\ldots$$

$$2\left(\frac{1}{6}\right)=2(0.161616\ldots)$$

$$\frac{2}{6}=0.3333\ldots$$

$$\frac{1}{3}=0.3333\ldots$$

2.

$$\frac{4}{9}=0.4444\ldots$$

$$\left(\frac{4}{9}\right)\div 4=(0.4444\ldots)\div 4$$

$$\frac{1}{9}=0.1111\ldots$$

Investigating the relationship between fractions and their equivalent decimal representations helps clarify to students that both representations stand for a single (rational) number on the number line. Since we teach students to perform computations with fractions and also to compute with decimals, performing these computations side-by-side with consistent results further reinforces this, and pulls these two representations into focus. In a way, we remove the mystery of these representations, showing that we can perform computations in any way we like; the results are equivalent. We have but to 'translate' the result into the representation we want.

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